Shallowness may be a major factor generating nutrient gradients in the Wadden Sea

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Abstract

Steep horizontal nutrient gradients are observed in the Wadden Sea. Along a transect from the German Bight to the mainland shoreline the concentrations of nitrate, ammonium, phosphate and silicate increase three to fivefold. This is surprising in view of the short residence times of the water in the basins of a few days only. In this paper, models are presented explaining the occurrence of such steep gradients. With decreasing depth and consequently lower mean tidal current velocities, phytoplankton removal by benthic suspension feeders and sedimentation of suspended particulate matter (SPM) increase. It is demonstrated that these factors alone lead to nutrient gradients. An assumed net inward transport of SPM amplifies these gradients. The considered models are 1D-equilibrium models with the distance from the shoreline as independent variable. The modelled processes are primary production, phytoplankton mortality, nutrient regeneration, horizontal mixing and transport of suspended particulate matter.

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1. Introduction

Nutrient concentrations in the surface-mixed layer are normally increasing from the centre of the North Sea towards the borders during all seasons, indicating the sink function of this shelf sea environment (Johnston and Jones, 1965; Johnston, 1973; Brockmann and Wegner, 1985; Brockmann et al., 1990). Data sets from winter and summer have been compiled since 1984 (Brockmann et al., 1999). This gradients continues beyond the open North Sea into the Wadden Sea and becomes even steeper. Along transects from the North Sea (German Bight) to the mainland, dissolved nutrient concentrations increase by a factor of 3 to 5 and more. The existence of these gradients for all nutrients is well known. Postma (1961a,b) has described very early a qualitative mechanism which could explain these gradients. Michaelis and Rahmel (1995) and Rahmel et al. (1995) gave a full year time series of weekly measured nutrient concentrations in the back-barrier Wadden Sea of the island Norderney. A review can be found in Liebezeit et al. (1997). Recently, Flöser et al. (2002) have measured these nutrient gradients with high accuracy in a number of different situations and places in the Eastfrisian Wadden Sea. In Colijn and Cadée (2003) some more references can be found. The steep gradi-
ents are remarkable because the residence time of the water masses in the Wadden Sea (behind the barrier islands) is only a few days. Hence, strong sources of dissolved nutrients have to exist in the inner parts of the Wadden Sea.

Several ecological models of the North Sea area exist. An important group of papers is connected to the ERSEM model (Ebenhöh et al., 1997; Lenhart et al., 1997; Ruardij et al., 1997; Allen, 1997, and others in the special issues ERSEM, 1995, 1997). For the authors of this paper the own experience with the ERSEM model is an important base for the present work. The modelling of the microbial loop of ERSEM was improved in Baretta-Bekker et al. (1998). A 3D-ecosystem model of the North Sea was presented by Moll (1997). This model and also the ERSEM model, however, cannot be used directly for the Wadden Sea. They are not constructed for shallow, tidally dominated basins, they do not take into account particulate transport, and they are too complex for a basic investigation of new process mechanisms. Generally, few ecosystem models extend into the Wadden Sea (Veldhuis et al., 1988; Ridderinkhof, 1988; Duwe and Hewer, 1982). Nutrient retention in the coastal wetland (Mitsch and Reeder, 1991; Wang and Mitsch, 2000) has some similarity to the nutrient dynamics in the Wadden Sea sediments discussed in this paper. In both cases a delayed nutrient cycling is considered. In contrast to the last mentioned papers, in the here presented model inward transport of suspended particles (detritus) and depth dependent mortality of phytoplankton control the nutrient cycle. These features are also not part of other recent models of shallow water ecosystems. Some address nutrient cycles and phytoplankton succession in these special and complex environments (Chapelle, 1995; Brawley et al., 2003). Chapelle uses three layers in the sediment, like ERSEM does, however, tidal movement does not occur in the modelled lagoon. Brawley et al. investigate phytoplankton production but do not explicitly model the nutrient dynamics. Bonnet and Poulin (2002) construct a vertical 1D-model of blooming cyanobacteria in a nutrient rich reservoir. The model of Angelini and Petere (2000) has three state variables only and does not consider the cycling of nutrients. Other recent papers concentrate mainly on the nitrogen cycle (Bahamon and Cruzado, 2003; Plus et al., 2003). The aquatic trophic network including fish is considered in Vasconcellos et al. (1997), Metzker and Mitsch (1997), Heymans and Baird (1999).

Most of the models focus either on the local nutrient cycle, including or excluding the microbial loop, on the phytoplankton succession, on the carbon flux network or on other aspects. Few take into account spatial aspects (beyond vertical structures). None of the models so far addresses the horizontal nutrient gradients in the Wadden Sea. Most of the models are box models with rather large boxes. They cannot be used to describe the interaction between biological processes and transport of water masses and particulate matter between shallow and deeper parts of the Wadden Sea and the adjacent open sea. The hypothesis of this paper is that this interaction creates the observed strong horizontal nutrient gradients. This is an hypothesis which is up to now not yet supported by models.

In this paper, a set of three strategic models will be developed to explain the gradients. The models centre on the spatial aspect, the temporal aspect is suppressed by assuming a steady-state situation. Then, ordinary differential equations with boundary conditions (at both ends) have to be solved. The independent variable is the distance x from the shore line. The emphasis is on the development of spatial structures (gradients). Therefore, the models in this paper are as much aggregated as possible, they contain only three state variables.

In another paper (Ebenhöh et al., 2003), the dimension of time is considered. Seasonal variability as well as daily and tidal variability are investigated. The spatial resolution is reduced to a small number of boxes. Then, differential equations with the independent variable time have to be solved. A more realistic ecosystem model is constructed, all four macro-nutrients (phosphate, nitrate, ammonium and silicate) are considered in parallel (as in the ERSEM model, 1997). Then comparison with observations (horizontal gradients as well as seasonal time series) is possible. This separation of the work parts was done because the three aspects, spatial structures, periodicity in time (seasonal, daily and tidal), and interacting multiple nutrient cycles are not directly dependent on each other. They need independent discussion and very different analysis and mathematical tools. The here presented paper concentrates on clarifying the principles of the gradient producing
mechanisms.

\[
\begin{align*}
\text{dissolved} & \xrightarrow{\text{production}} \text{particulate} & \xrightarrow{\text{loss}} \text{benthic} \\
\text{efflux} & \xrightarrow{\text{dissolved}} \\
\end{align*}
\]

The model describes the cooperation of the nutrient cycle with the transport of dissolved and suspended matter. The nutrient cycle is condensed to three physically different states dissolved, particulate and benthic. Therefore, a minimum of three state variables is necessary:

- dissolved nutrients \( N \) (in the water column)
- particulate nutrients \( P \) (in phytoplankton and pelagic detritus)
- benthic nutrients \( B \) (in form of benthic dissolved nutrients and detritus)

This cycle is driven by the primary production. The loss processes are due to sinking and grazing by benthic suspension feeders. Only losses are considered which go into the sediment. The regeneration of the nutrients in the benthos is only implicitly contained in the cycle. The efflux of regenerated and dissolved nutrients from the benthic system into the water column depends on the pore water concentration and other factors. The three processes connecting the three state variables are abbreviated in the formulas with prod, loss and effl. The cycle interacts with horizontal transport processes: mixing acts on \( N \) and \( P \), and the transport of resuspended benthic matter acts on \( B \). The latter transport in due to the asymmetric tidal movement. In the model the cooperation of the cycle processes and the transport processes produces the steep increase of the pelagic nutrient concentration \( N \) toward the shoreline. In the following sections three equilibrium models are analysed which successively take into account the shallowness gradient, mixing and asymmetric particulate transport:

(M1) Isolated water columns with given different depths \( H \)

All processes depend on the water depth \( H \) in different ways. Shallowness is an important factor. Hence, the equilibrium distribution of the nutrients over the three state variables in the cycle depends on \( H \) even without horizontal transport. This will be demonstrated with this simple model version.

(M2) The nutrient cycle of model (M1) is extended by mixing of \( N \) and \( P \) along a transect from the open Sea (German Bight, North Sea) through the Wadden Sea to the mainland shoreline (from deep to shallow)

The dashed lines indicate transport by mixing. The net effect of mixing is directed, it follows the concentration gradient. Dissolved nutrients \( N \) are mixed outward, because the concentrations are higher in the near shore areas. These losses are balanced by an inward mixing of phytoplankton and (pelagic) particulate matter \( P \). This reversal of the concentration gradient of \( P \) is an artefact of this model (M2), it is unrealistic. Therefore, the model has to be further refined:

(M3) To repair this artefact of model (M2), it is extended to take into account a directed transport of suspendable benthic material due to “tide-induced residual particulate matter transport”.

Now, the nutrient losses to the outer Wadden Sea are compensated by import of benthic material \( B \).

Dronkers et al. (1990) present a quantitative analysis of the transport dynamics, first postulated by Postma (1961a,b), that are based on the existence of an ebb-flood asymmetry. The essential assumption in Postma’s theory is that the current speed necessary to resuspend settled material from the bottom is larger than the current speed necessary to keep this material in suspension. When the tidal and geometrical conditions experienced by particles during ebb and flood are not symmetrical, then the sedimentation of particulates at HWS (high water slack) and LWS as well as the resulting relative displacements of these
particulate masses will be different. The net transport which results, is called "tide-induced residual particulate matter transport". It points toward the shore line.

2. The isolated water column model (M1)

The equations for the isolated model have no transport terms (M1):

\[
\begin{align*}
0 &= \dot{N} = \text{effl} - \text{prod}(H) \\
0 &= \dot{P} = \text{prod}(H) - \text{loss}(H) \\
0 &= \dot{B} = \text{loss}(H) \times H - \text{effl}
\end{align*}
\]

The steady state is considered, hence, all time derivatives vanish. Obviously, the equilibrium demands

\[
\begin{align*}
\text{prod}(H) &= \text{loss}(H) = \text{effl} \\
(1)
\end{align*}
\]

The interesting feature in this model is that all processes may depend on \(H\), and hence, also the equilibrium concentrations \(N\), \(P\) and \(B\) will do so. If all variables are taken in the same "currency" (say nitrogen \(N\)), the total nutrient content of the system (vertically integrated) is

\[
(N + P)H + B
\]

and it is conserved in a closed system. The pelagic variables \(N\) and \(P\) have units (mmol N m\(^{-3}\)) and the benthic variable \(B\) has the unit (mmol N m\(^{-2}\)), while \(H\) is measured in meter and converts pelagic and benthic units. To be able to compare systems with different depths, one has to keep the total nutrients in the systems comparable. It cannot be the same, because the depths change, but it also should not be simply proportional to the depth, because the benthic system contains nutrients. Hence, it is reasonable to select an effective benthic layer \(d_B\) and to keep the nutrient density constant in \(H + d_B\):

\[
(N + P)H + B = N^{0\text{th}}(H + d_B)
\]

In the systems (M2) and (M3) in the following sections, this condition (3) has to be substituted by boundary conditions. Nutrients can then be exchanged between locations with different depths \(H\).

For the process descriptions, the following assumptions are made:

\[
\begin{align*}
\text{prod} &= r_{NP}(H)g(N)P \\
\text{loss} &= r_{PB}(H)P \\
\text{effl} &= r_{BN}(B - N d_B)
\end{align*}
\]

The production "prod" depends on the nutrient concentration via \(g\) but not on time, because seasons are not considered. The rate constant \(r_{NP}\) (maximal specific productivity, in m\(^{-3}\) per day) is depth dependent, because it contains implicitly the light climate. The model is vertically integrated and the average light is reduced if \(H\) is larger, and further, in shallow waters the turbidity is generally higher. However, the model works well with a depth independent parameter \(r_{NP}\). The loss term loss of phytoplankton shall depend on depth \(H\). This is a central assumption in this paper. The efflux effl of benthic nutrients is proportional to the concentration gradient between the dissolved nutrients in the water column and in the pore water. Here, clearly, a limitation of the model is obvious: The benthic nutrient \(B\) contains a dissolved as well as a particulate fraction. Calculating a "pore water concentration" is fictitious, but also the use of an "effective benthic layer" \(d_B\). However, to keep the model as simple as possible, these fractions are not separated. The equilibrium concentrations can then be calculated from the equilibrium condition (2):

\[
g(N) = \frac{r_{PB}(H)}{r_{NP}(H)} \quad \text{or} \quad P = 0
\]

\[
N = g^{-1} \left( \min \left( \frac{r_{PB}(H)}{r_{NP}(H)} g(N^{0\text{th}}), \frac{1}{g(N^{0\text{th}})} \right) \right)
\]

Here it proves of advantage that in (4) the efflux effl was constructed in such a way, that for \(P = 0\) the equilibrium concentrations according to (3) are \(N = N^{0\text{th}}\) and \(B = N^{0\text{th}} d_B\). The other equilibrium concentrations follow:

\[
P = (N^{0\text{th}} - N) \left( \frac{r_{BN}}{r_{BN} + r_{PB}(H)} \left(1 + \frac{d_B}{H} \right) \right)
\]

\[
B = \frac{r_{PB}(H)}{r_{BN}} HP + N d_B
\]

For the visualization of this result, a Monod function for \(g\) and simple functional dependences of the rates on the depths have been chosen:
Table 1
Parameters and parameter values used in the simulations. Given is a standard set of parameters; modified values are given in brackets.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{NP}$</td>
<td>Primary production rate</td>
<td>Per day</td>
<td>0.3</td>
</tr>
<tr>
<td>$r_{PB}$</td>
<td>Mortality/sedimentation rate at $H_{NS}$</td>
<td>Per day</td>
<td>0.05</td>
</tr>
<tr>
<td>$\bar{r}_{PB}$</td>
<td>Nutrient efflux rate</td>
<td>Per day</td>
<td>0.05</td>
</tr>
<tr>
<td>$K$</td>
<td>Michaelis constant for primary production</td>
<td>(mmol N/m$^{-3}$)</td>
<td>1</td>
</tr>
<tr>
<td>$N_{tot}$</td>
<td>Total nutrient concentration</td>
<td>(mmol N/m$^{-3}$)</td>
<td>30</td>
</tr>
<tr>
<td>$d_{B}$</td>
<td>Effective benthic layer</td>
<td>m</td>
<td>0.2</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Dependence of mortality on depth $r_{PB} = r_{PB}(H_{NS}/H)^{\alpha}$</td>
<td></td>
<td>1 (0.5, 0)</td>
</tr>
<tr>
<td>$x_{NS}$</td>
<td>Distance shore line to North Sea (German Bight)</td>
<td>km</td>
<td>20</td>
</tr>
<tr>
<td>$H_{NS}$</td>
<td>Depth at$\alpha_{NS}$ geometry: $H_{NS}, W_{NS}$ (see Fig. 2)</td>
<td>m</td>
<td>10</td>
</tr>
<tr>
<td>$k_{0}$</td>
<td>Diffusion constant for horizontal mixing</td>
<td>km$^{2}$ per day</td>
<td>5 (10)</td>
</tr>
<tr>
<td>$H_{1}$</td>
<td>Stronger mixing in shallow water (see text)</td>
<td>m</td>
<td>0.2</td>
</tr>
<tr>
<td>$L_{0}$</td>
<td>Distance of age reset to zero</td>
<td>km</td>
<td>10 (15)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Source parameter for benthic matter flux</td>
<td>(mmol N/m$^{3}$) per day</td>
<td>20 (0)</td>
</tr>
</tbody>
</table>

\[ g(N) = \frac{N}{N + K} \quad r_{PB}(H) = r_{PB} \left( \frac{H_{NS}}{H} \right)^{\alpha} \]

A reasonable parametrization not too far from reality is given in Table 1. The value for $r_{PB}$ (0.05 per day) corresponds to a sinking velocity of 0.5 m per day in a 10 m water column ($H_{NS}$). In the standard calculation, the power $\alpha$ is 1. Only for comparison and demonstration, $\alpha$ was set to 0 and 0.5.

The calculation is easy because no differential equations have to be solved. Results are shown in Fig. 1. It can be clearly seen that for $\alpha = 1$ the nutrient concentrations rise strongly toward the shore line. The phytoplankton concentrations decline and reach zero if the

Fig. 1. Equilibrium distribution of the nutrients over the three state variables in dependence on the depth $H$ of the water column in model (M1). The power $\alpha$ describes the depth dependence of the mortality rate $r_{PB}(H)$. 

\[ \text{Nutrient concentration as function of depth H} \]

\[ \text{Phytoplankton density as function of depth H} \]

\[ \text{Benthic nutrients} \]

\[ \text{Fraction of nutrients in the benthos} \]
mortality exceeds production. This would not be the case if \( K \) would be removed (prod = \( r_{NP} NP \)). Then, there is not an upper limit for the specific growth rate. If, on the other hand, one would select for \( K \) a more realistic value smaller than 1 (mmol Nm\(^{-3}\)) the simulation results would look similar but still more compressed toward the shore line. This is not the case for the models (2) and (3) where diffusion spreads out and smoothes the gradient.

3. A model with mixing (M2)

In this section, an independent continuous spatial variable \( x \) is introduced in the model. It is the distance from the shore line \( x = 0 \) and runs through the Wadden Sea to the open North Sea (the German Bight). The distance to the outer boundary is \( x = X_{\text{NS}} \). Along the \( x \)-axis depth \( H(x) \), horizontal width \( W(x) \) and vertical cross-section \( Q(x) \) change. For simplicity, we assume a rectangular cross-section:

\[
Q(x) = W(x)H(x)
\]

The differential Eq. (1) have to be extended by transport terms, describing mixing along the \( x \)-axis. These terms are taken as “diffusion” terms with a diffusion constant \( \kappa \). Hence, we have to deal with \( x \)-derivatives. A positive flux \( Q \) over the cross-section leads us back to (mmol N m\(^{-2}\)) per day which are the units of prod, loss and effl/H. 

The time derivative in the equation system vanish because in this paper we consider only stationary states. Then the system can be rewritten into the simple form (ode system of second order)

\[
(\kappa QN)' = Q(\text{prod} - \text{loss}) = - (\kappa QP)'
\]

Along the \( x \)-axis depth \( H(x) \), horizontal width \( W(x) \) and vertical cross-section \( Q(x) \) change. For constant \( \kappa \) one has to handle a singularity. We can avoid the problem either by keeping the depth \( H \) artificially finite at

\[
A(x) = \frac{L_0^2 - x^2}{2\kappa}, \quad x < L_0
\]

For constant \( \kappa Q \) as rough approximation, the solution of (10) is simply

\[
A(x) = \frac{L_0^2 - x^2}{2\kappa}, \quad x < L_0
\]

For varying cross-sections, \( Q \) numerical calculations have to be carried out (see below).

3.1. Boundary conditions

For solving the differential equations, one has to specify boundary conditions at \( x = X_{\text{NS}} \) and at \( x = 0 \). This, however, is not trivial. Because on the shore, boundary the depth vanishes

\[
H(x) \to 0 \quad \text{for} \quad x \to 0
\]

one has to handle a singularity. We can avoid the problems either by keeping the depth \( H \) artificially finite at
\( x = 0 \) or by choosing an \( x \)-dependent diffusion constant which goes to infinity in such a way that \( \kappa Q \) stays finite at the shore line:

\[
x(s)Q(x) \to \kappa Q(0,.) > 0 \quad \text{for} \quad x \to 0
\]

In agreement with \( \kappa Q(0,.) > 0 \), we define \( \kappa \) as

\[
x(s) = \frac{H(\kappa + H)}{H(1)}
\]

Four boundary conditions (B1)-(B4) have to be posed (2 ode of second order). We consider a closed system without nutrient export or import. This implies

no transport over the North Sea boundary:

\[ N'(\xi_{NS}) = P'(\xi_{NS}) = 0 \quad \text{B1+B2} \]

no transport over the shore boundary:

\[ N(0) = P(0) = 0 \quad (\kappa Q(0,.) > 0) \quad \text{B3} \]

The latter counts only as one condition, because \( \kappa Q(N + r) \) is constant along \( x \), and this constant is already defined as \( 0 \) in conditions (B1 + B2). Due to the fourth boundary condition the total nutrient mass is defined. Technically, it is most convenient to fix the phytoplankton density at the North Sea boundary. There the (local) equilibrium values without mixing according to model (M1) are

\[ N_{NS} = g^{-1} \left( \frac{r_{NS}(H_{NS})}{r_{NP}} \right) \]

\[ P_{NS} = (N_{NS} - N_{BN}) \left( \frac{r_{BN} + r_{NP}(H_{NS})}{r_{NP}} \left( 1 + \frac{d_{h}}{H_{NS}} \right) \right) \]

with \( H_{NS} = H_{NS(N)} \). The equilibrium values of (M1) cannot both be assumed at \( x = \xi_{NS(0)} \). Therefore only near local equilibrium at the North Sea boundary can be selected as fourth boundary condition:

\[ P(\xi_{NS}) = P_{NS} - N(\xi_{NS}) = N_{NS} + r_{NP} \quad \text{B4} \]

Technically the system is solved as initial value problem with initial values (B1 + B2 + B4) at \( x = \xi_{NS} \) for a selected \( x_{NS} \). The integration runs down to \( x = 0 \) and is repeated with a modified parameter value \( r_{NP} \) until at \( x = 0 \) the solution fulfills the boundary condition (B3).

For the numerical simulations, the same set of parameters as in the previous section has been used. In addition, assumptions on the geometry of the model Wadden Sea have to be made. The chosen functions

\[ \kappa(x) \rightarrow \infty \quad \text{Q} \]

\[ \kappa(x) \rightarrow 0 \quad \text{W} \]

\[ \kappa(x) \rightarrow \text{finite at the shore line:} \]

\[ \kappa(x) Q(\xi_{NS}) \]

\[ \kappa(x) Q(\xi_{NS}) \]

are given in Fig. 2. The length of the distance \( x \) runs from the shoreline (0 km) to the open German Bight (20 km). The inlet between the barrier islands lies between 10 and 15 km. There the width \( W \) is small and the depth \( H \) is large. The tidal flats lie at distances less than 10 km. There the depth decreases and the width increases toward the land. \( Q \) is the vertical cross-section \( (Q = H \times W) \) at distance \( x \). This geometry is used in the models (M2) and (M4).

\[ H(x) \] and \( W(x) \) are crude approximations to the Wadden Sea basins on the lee side of the East Frisian islands. They are given in Fig. 2. The length of the distance \( x \)-interval was fixed to \( x_{NS} = 20 \text{ km} \), the boundary depth was chosen to be \( H_{NS} = 10 \text{ m} \) (Table 1). The distance interval \( 15 < x < 10 \text{ km} \) corresponds to the inlet between the islands (narrow and deep), the distance below 10 km represents the tidal flats (increasingly shallow).

In Fig. 3, some numerical results of model (M2) are given. With increasing mixing parameter \( \kappa \) the gradients become less steep, but—especially for the nutrients—they do not vanish. The concentration of benthic nutrients is reduced in areas with shallow water (in this model). The fraction of the total nutrients contained in the benthic layer increases with the shallowness of the water column (not shown).

With the same geometry and transport parameters the age of water masses can be calculated numerically according to (10). The age was reset to 0 at a distance of \( L_0 = 10 \text{ km} \) (15 km) from the shore line (Fig. 3, lower right). Then, for a mixing constant of \( \kappa = 10 \text{ km}^2/\text{day} \) a maximal age at the shore line of 2.8 days (6.5 days) was found, and an average age of 1.7 days (3.6 days). With \( \kappa = 5 \text{ km}^2/\text{day} \) the maximal age is 5.6 days (12.5 days) and the average age 3.2 days (6.8 days). The average age \( A(x) \) weighted with the cross-section is lower than the maximal age.
Fig. 3. Results of model (M2). (Upper graphs) Pelagic nutrient and phytoplankton concentrations as a function of the distance \(x\) from the shoreline. In an insert the water depth \(H\) is given for orientation (see Fig. 2). With increasing mixing parameter \(\kappa\) the gradients become less steep. (Lower left) The benthic nutrient concentrations decrease toward the shore line (see text). (Lower right) The age \(A(x)\) of the water masses in dependence of the distance \(x\) from the shore line. The age is reset to zero at 10 resp. 15 km. The average ages \(\bar{A}(x)\) are indicated by horizontal lines.

\[
A(0) \text{ because the "old" water belongs to shallow areas and gets low weight:}
\]

\[
\bar{A}(x) = \frac{\int_0^x A(z)Q(z) \, dz}{\int_0^x Q(z) \, dz}
\]

4. A model with mixing and transport of benthic material (M3)

Mixing (Fig. 3) reduces the gradients. However, with moderate mixing at least the nutrient gradients remain impressive. The main drawback of the model (M2) is the dilution of the benthic system with decreasing depth. This is unrealistic. The prevalence of muddy tidal flats in the inner part of the Wadden Sea indicates that there is an inward transport of fine particulate matter, including detritus (containing nutrients), which enriches the sediments with nutrients in nearshore areas. Therefore, a third model (M3) has been constructed, which takes into account this additional transport mechanism for resuspendable benthic material (silt, detritus). This model, at the same time, makes the nutrient gradients steeper. As mentioned earlier the benthic "nutrient" \(B\) in this paper does not differentiate between dissolved (nutrients in the pore water) and particulate (nutrients contained in benthic detritus and organisms). This was done to keep the model as simple as possible, as it is a conceptual model. The "tide-induced residual particulate matter transport" considered here moves the particulate fraction of \(B\) which in the following is denoted as benthic detritus.

Model (M3) is constructed by extending the equation system (7) from model (M2). Now an inward flux
$j_B$ of benthic material is taken into account. Its sources are denoted as $\sigma_B$:

$$
0 = N = \frac{\text{effl}}{H} - \text{prod} - \frac{1}{Q}(\text{effl})' \sigma_B = (j_B)' \\
0 = P = (\text{prod} - \text{loss}) - \frac{1}{Q}(\text{loss})' \sigma_B = (j_B)' \\
0 = B = H \times \text{loss} - \frac{1}{W}(\text{loss})' \sigma_B = (j_B)'
$$

(12)

The transport $j_B$ across the width $W(x)$ of the sediment surface has units (mmol N)$^2$ per day. This flux transports matter inward with sources in the outer parts of the Wadden Sea and sinks in the inner parts. It is not proportional to the gradients of $B$, it can even act against the concentration gradient. The mechanism is resuspension by the tidal currents and resedimentation of benthic material. Resuspension takes place at MW (mid water) when the tidal currents are fastest, resedimentation occurs around LW and HW (low and high water), when the current velocity is nearly zero (slack). Asymmetries arise in two ways:

(1) The vertical velocity profiles for ingoing and out-going tides differ. Ingoing tides have a more homogeneous profile and, hence, higher ground velocities. (2) At HW the water is spread over the tidal flats and thus has a lower average depth than at LW. Therefore, a larger fraction of the suspended matter can settle out at HW than at LW. The flux $j_B$ reflects the net effect of this tidal asymmetry. Mass conservation requires that $j_B$ vanishes at both boundaries. This implies that the integral over the sources $\sigma_B$ vanish:

$$
\int_0^{x_{NS}} \sigma_B \, dx = j_B(x_{NS}) - j_B(0) = 0
$$

$$
\int_0^{x_{NS}} \left(\text{loss} - \frac{\text{effl}}{H}\right) \, dx = 0 \quad (Q = W \times H)
$$

(13)

Fig. 4. Comparison of the simulation results of model (M3) with and without benthic transport (l). The mixing parameter in both cases is $\kappa = 10 \text{m}^2 \text{d}^{-1}$. 

\begin{itemize}
  \item \text{Nutrient concentrations} $\kappa = 10 \text{m}^2 \text{d}^{-1}$
  \item \text{Phytoplankton concentrations}
  \item \text{Sediment: Input and output}
\end{itemize}
The same integral is connected to the pelagic fluxes:

\[
0 = \int_{0}^{x_{NS}} Q \left( \frac{\text{loss} - \text{effl}}{H} \right) \, dx = \int_{0}^{x_{NS}} (\kappa Q(N + P)') \, dx
\]

\[
\kappa Q(N + P)'(0) = 0
\]

In the case of \( j_B = 0 \) (M2) the expression \( \kappa Q(N + P)' \) is constant and vanishes everywhere because it vanishes at one boundary. Now it is no longer constant, however, it has identical values at both interval ends, which again are zero. Hence, the same boundary conditions (B1)–(B4) as for (M2) can be used for (M3). The difference between the two systems is that \( \text{effl}/H \) and \( \text{loss} \) are not equal. Rather, they differ due to the source of the benthic flux by \( \sigma_B Q(\ldots) \):

\[
\frac{1}{Q} (\kappa Q(N + P)') = \text{prod} - \frac{\text{effl}}{H}
\]

\[
\frac{1}{Q} (\kappa Q(P)') = (\text{loss} - \text{prod})
\]

\[
\text{effl} = H \times \text{loss} - \frac{\sigma_B(x)}{W}
\]

The new system (M3) also contains two differential equations of second order. For numerical calculations one has to specify \( \sigma_B(x) \). We choose

\[
\sigma_B(x) = \begin{cases} 
-\lambda & 1 < x < 15 \text{ km} \\
\lambda & 0 < x < 1 \text{ km}
\end{cases}
\]

Any other choice of similar character will suffice also. There must be a large interval in the outer Wadden Sea with moderate sources of benthic material (the inward flux is mathematically negative, and hence the “source” carries a minus sign) and a short near shore interval with sinks for the material. The parameter \( \lambda \) has units \((\text{mmol N})\text{m}^{-1}\) per day. A reasonable value for \( \lambda/W \) at \( x = 10 \text{ km} \) \((H = 5 \text{ m}, W = 10 \text{ km})\) will be about a quarter of the benthic input loss \( \times H \):

\[
\frac{\lambda}{W} \approx \frac{1}{4} \text{reg}(H) P(x) \approx 2 \text{ (mmol N)}\text{m}^{-2} \text{ per day}
\]

This means, a fraction of 25% of the daily input feeds the asymmetric net inward flow of benthic matter. This is a tiny fraction of the benthic detritus stock,
because the turn over time (of benthic detritus and \( B \) in the model) is about 20 days. Simulation results are presented in Fig. 4. It can be seen that the nutrient gradient became much steeper due to the incorporation of particulate transport in the model. The increase of benthic nutrients \( B \) (including detritus) is only very moderate. This is because most of the surplus is rapidly remineralized. Hence, the effect can best be seen in the pelagic concentrations. This would be quite different if a conservative substance like silt is considered. With the same transport mechanism, the silt accumulation close to the shoreline turns out to be enormous (not shown), witnessed by the high clay content in the near shore Wadden Sea.

5. Summary and discussion

The models presented here are strategic models. They clearly demonstrate that the explanation for the steep nutrient gradients in the Wadden Sea can be found in the biologically driven cycling of nutrients between dissolved and particulate and pelagic and benthic stages, with tide-induced residual sediment and SPM transport providing the raw material. The nutrient cycle depends on the water depth, with the benthic subsystem being obviously more important in shallower waters. Another modelled feature is the “cooperation” of different parts of the Wadden Sea. Due to the steep nutrient gradients and the strong mixing, nutrients leave the Wadden Sea in dissolved form. Because the model is a steady-state model the outward transport of dissolved nutrients is perfectly compensated by the net inward transport of nutrients in particulate form as SPM. This can be pelagic or benthic detritus. The model underscores the central role of SPM as carrier of nutrients in meso- and macro-tidal estuarine deposition areas. It also fits in the recently started discussion on modelling benthic-pelagic coupling (ERM, 1995). A comparison of the model with real data can only be preliminary. The model should be extended to include more state variables (Benchmark et al., 2003) and all four macro-nutrients like in ERSEM (1995, 1997), and the benthic nutrient dynamics should differentiate between particulate and dissolved states. Nevertheless, as an example in Fig. 5, a set of measurements by Fleser et al. (2002) from July 2001 are shown and compared with scaled model results. In the strategic models of this paper the “nutrient” was not specified, hence, the results can be compared with all macronutrients, if factors are used. The result of model (M3) is multiplied with 2 (silicate, ammonium) and 0.35 (phosphate). Nitrate is not shown because it is low in July, however steep nitrate gradients exist in other seasons.

References


